Parametric Functions

Objectives: Review parametric functions; Perform differential Calculus operations on parametric curves.

Definition of Parametric curve:

Ex 1. Sketch the curve described by the parametric equations

$$x = t^2 - 4 \quad \text{and} \quad y = \frac{t}{2}, \quad -2 \leq t \leq 3.$$

Ex 2. Finding a rectangular equation that represents the graph of a set of parametric equations is called **eliminating the parameter.**

$$x = t^2 - 4 \quad \text{and} \quad y = \frac{t}{2}, \quad -2 \leq t \leq 3.$$

Ex 3. Eliminate the parameter.

a. $x = 2\cos t \;\; y = 2\sin t \;\; 0 \leq t \leq 2\pi$ **b.** $x = \sqrt{\frac{t}{2}} \quad y = t - 3$

Smooth curve:

Parametric form of the derivative:

Ex 4. Find dy/dx for the smooth curve given by $x = \sin(t)$ and $y = \cos(t)$. Then find the equation of the tangent at $\frac{\pi}{4}$

Second derivative:

Ex 5. Find the second derivative in terms of t if $x = t - t^2$ and $y = t - t^3$

You try. Graph indicating the direction traced. Find the slope and concavity at the point (2,3)

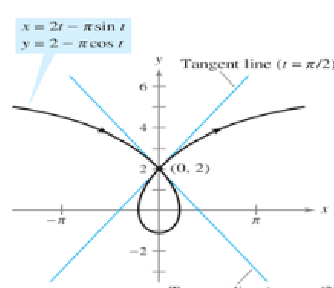
$x = \sqrt{t} \quad y = \frac{1}{4}(t^2 - 4)$

You try Graph indicating the direction traced. Find where the graph has a horizontal tangent. Then find where the second derivative is 0 or undefined.

$x = t^2 - 5 \quad y = 2\sin t \quad 0 \le t \le \pi$

The **prolate cycloid** given by crosses itself at the point (0, 2), as shown. Find the equations of both tangent lines at this point.

$$x = 2t - \pi \sin t \qquad \text{and} \qquad y = 2 - \pi \cos t$$



This prolate cycloid has two tangent lines at the point (0, 2).

For the cycloid defined by $x = t\sin(t)$ and $y = 1-\cos(t)$, determine the concavity at $t = \pi$